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Influence of Intrinsic Decoherence in the Presence of Stark Shift on Nonclassical Properties of the Two-Mode JCM

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We study the influence of intrinsic decoherence in the presence of Stark shift for the two-mode Jaynes–Cummings model (JCM). An analytic solution of the Milburn equation for the multiquant two-mode JCM Hamiltonian is obtained. We use this solution to investigate the influence of intrinsic decoherence and Stark shift on nonclassical properties of the system, for the resonant and the off-resonant cases. We compare the behavior of the system in the case of having a coherent superposition state and a statistical mixture of coherent states as an initial field.

KEY WORDS: intrinsic decoherence; Stark shift; two-mode JCM.

1. INTRODUCTION

The motion of the center-of-mass (CM) of ultracold trapped ions has to be dealt with quantum mechanically (Diedrich *et al.*, 1989; Monroe *et al.*, 1995a). Laser irradiation is used to monitor the ion's internal and external degrees (Blockley *et al.*, 1992, 1993; Cirac *et al.*, 1993a,b; de Matos Filho and Vogel, 1994, 1996a,b; and Vogel and de Matos, 1995). On the other hand, ion trap quantum computation, first introduced by Cirac and Zoller (1995), is a potentially powerful technique for the storage and manipulation of quantum information. Recently, much progress has been made in the preparation, manipulation, and measurement of quantum states of the center-of-mass vibrational motion of a single atom experimentally (Itano *et al.*, 1997; King *et al.*, 1998; Leibfried *et al.*, 1996; Meekhof *et al.*, 1996; Monroe *et al.*, 1995b,c, 1996; Roos *et al.*, 1999) and theoretically (Bardoff *et al.*, 1996; Cirac and Zoller, 1995; de Matos Filho and Vogel, 1994, 1996; D'Helon and Milburn 1995; Gerry *et al.*, 1997; Gou *et al.*, 1996a,b,c; Gou and Knight, 1996;

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Kneer and Law, 1998; Poyatos *et al.*, 1996a,b; Steinbach *et al.*, 1997; Wallentowitz *et al.*, 1997; Wallentowitz and Vogel, 1995), which are not only of fundamental physical interest but also of particular use for sensitive detection of weak signals (Bachor, 1998) and quantum computation in ion trap (Cirac and Zoller, 1995; Monroe *et al.*, 1995b).

Models have been constructed to describe a two-level ion undergoing quantized vibrational motion within a harmonic trapping potential and interacting with a classical light field (Blockley *et al.*, 1992, 1993; Cirac *et al.*, 1992, 1993a,b,c). It has been pointed out that the dynamics of a trapped ion can be described by a Hamiltonian similar to a Jaynes–Cummings model (Jaynes and Cummings, 1963) or its generalizations under certain regimes (Buzek *et al.*, 1997; de Matos Filho and Vogel, 1994, 1996; Gou *et al.*, 1996a,b,c; Steinbach *et al.*, 1997; Vogel and de Matos Filho, 1995). Despite these heroic experimental achievements, the quantum motion of a single atom is obviously limited by sources of decoherence. Decoherence arises from random and unknown perturbations of the Hamiltonian. If these perturbations cannot be followed exactly, experiments must average over them. This leads to an effective irreversible evolution of the atom and a suppression of coherent quantum features through the decay of off-diagonal matrix elements of the density operator in some basis. Complementary to the decay of off-diagonal matrix elements, noise is added to conjugate variables. This can appear as a heating of the atom if noise is added to the momentum variable. On the other hand, there has been increased interest in the decoherence problem in quantum mechanics because of its possible application in quantum measurement processes and quantum computers (Shore, 1995; Chuang and Yamamoto, 1997).

The intrinsic decoherence approach has been proposed and investigated in the framework of several models (Caves and Milburn, 1987; Diosi, 1989; Ellis *et al.*, 1989, 1990; Ghirardi *et al.*, 1986). In particular, Milburn (1993) proposed a simple intrinsic decoherence model based on an assumption that on sufficiently short time steps the system evolves in a stochastic sequence of identical unitary transformations. This assumption modifies the von Neumann equation for the density operator of a quantum system through a simple modification of the usual Schrödinger evolution equation. The off-diagonal elements of the density operator in Milburn's model are intrinsically suppressed in the energy eigenstate basis, thereby intrinsic decoherence is realized without the usual dissipation associated with the normal decay. The decay is entirely of phase dependence only. Free evolution of a given quantum system has been discussed early (Milburn, 1993) but investigations of interacting subsystems follow (Chen and Kuang, 1994; Hessian, 2002; Kuang *et al.*, 1995; Kuang and Chen, 1994; Moya-Cessa *et al.*, 1993; Obada *et al.*, 1998, 1999a).

Decoherence due to normal decay is often said to be the most efficient effect in physics, to a point where observation comes too late after the effect has reached completion (Omne's, 1997). The effect in action has been observed in quantum optics where the decoherence phenomena transforming a Schrödinger-cat into a statistical mixture was observed while unfolding (Brune *et al.*, 1996). There has been considerable interest in the properties of the so-called superposition states of light (SS) involving superpositions of coherent states with strongly differing amplitude (Buzek *et al.*, 1992; Buzek and Knight, 1991; Janszky and Vinogradov, 1990; Mandel, 1986; Schleich *et al.*, 1991; Vidiella-Barranco *et al.*, Wodkiewicz *et al.*, 1987). One particularly interesting case is the superposition of two coherent states of fixed amplitude but opposite phase (Buzek *et al.*, 1992; Buzek and Knight, 1991; Janszky and Vinogradov, 1990; Mandel, 1986; Schleich *et al.*, 1991; Vidiella-Barranco *et al.*, 1992). Due to the quantum interference, the properties of such a superposition are very different from the properties of the constituent states (coherent states), as well as from the incoherent superposition or statistical mixture (SM) of coherent states.

The purpose of this work is to study the influence of intrinsic decoherence in the presence of Stark shift for the two-mode JCM. In particular, we are interested in the decay of macroscopic coherences induced by intrinsic decoherence by comparing the dynamics of SS and SM states. We will obtain an exact solution of the Milburn equation for the multiquant JCM. We use this solution to study the influence of the intrinsic decoherence on nonclassical properties in the presence of Stark shift in the resonant or the off-resonant cases. Finally, conclusions are provided. This paper is organized as follows: In section 2, we obtain an exact solution of the Milburn equation for the multiquanta JCM and give the explicit expression of this solution in the two-dimensional basis of the particle. Section 3 is devoted to an investigation of the influence of the intrinsic decoherence on nonclassical properties either in the resonant or the off-resonant cases. Finally, some concluding remarks are provided.

2. EXACT SOLUTION OF THE MILBURN EQUATION

We consider a quantum system described by the density operator $\rho(t)$. In standard quantum mechanics, dynamics of the system is governed by the evolution operator $\hat{U}(t) = \exp[-\frac{i}{\hbar}t\hat{H}]$, where $\hat{H}$ is the Hamiltonian describing the system. Milburn assumed (Milburn, 1993) that on sufficiently short time steps the system does not evolve continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformations. On the basis of this assumption, he has derived the equation for the time evolution density operator $\rho(t)$ of the quantum system (Milburn, 1993)

$$\frac{d}{dt}\hat{\rho}(t) = \gamma\left\{\exp\left[-\frac{i}{\hbar\gamma}\hat{H}\right]\hat{\rho}(t)\exp\left[\frac{i}{\hbar\gamma}\hat{H}\right] - \hat{\rho}(t)\right\} \tag{1}$$

where γ is the mean frequency of the unitary time step. This equation formally corresponds to the assumption that on sufficiently short time steps the system evolves with a probability $p(\tau) = \frac{\tau}{2\gamma}$. Obviously, the generalized Eq. (1) alters the Schrödinger dynamics. It reduces to the ordinary von Neuman equation for the density operator in the limit $\gamma \to +\infty$. Expanding Eq. (1) to first order in γ^{-1}, the following dynamical equation is obtained:

$$\frac{d}{dt}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{1}{2\hbar^2\gamma}[\hat{H}, [\hat{H}, \hat{\rho}]] \tag{2}$$

which is the Milburn equation that we shall study below. This equation has been solved for a harmonic oscillator and a precessing spin system (Milburn, 1993) the simple JCM (Chen and Kuang, 1994; Kuang and Chen, 1994; Moya–Cessa *et al.*, 1993), the resonant multiphoton JCM (Kuang *et al.*, 1995), and the nondegenerate two-mode JCM (Obada *et al.*, 1998, 1999a; Hessian, 2002). In what follows we shall consider the exact solution of this equation for the two-mode JCM in the presence of Stark shift with a detuning parameter in either a superposition state (SS) or a mixture state (SM).

We shall consider a Hamiltonian model that consists of two modes interacting with a three-level particle (atom or trapped ion) via Raman transition. We consider the nondegenerate case in which pairs of photons with two different frequencies are created or annihilated. The quantized radiation field is considered in the rotating wave approximation frame taking into account the effect of Stark shift. The atomic levels have identical parities such that each dipole is coupled with different modes of the field and to the set of intermediate states. If we assume that the intermediate states do not admit dipole transitions between themselves and the interaction field modes are far off-resonance from those intermediate states, then the particle can be seen as an effective two-level system by means of adiabatic elimination of the intermediate state (Alsing and Zubairy, 1987; Li and Peng, 1995; Obada *et al.*, 1999b). The Hamiltonian for the system, in the rotating wave approximation, is written as

$$\begin{aligned}\hat{H} &= \frac{\omega_0}{2}\hat{\sigma}_z + \sum_{j=1}^{2}\omega_j\hat{a}_j^\dagger\hat{a}_j + \hat{a}_1^\dagger\hat{a}_1\beta_1|g\rangle\langle g|\hat{a}_2^\dagger\hat{a}_2\beta_2|e\rangle\langle e| \\ &\quad + \lambda\left(\hat{a}_1^{\dagger k_1}\hat{a}_2^{k_2}\hat{\sigma}_- + \hat{a}_1^{k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_+\right) \\ &= \omega_1\left[\hat{n}_1 + \frac{k_1}{2}(I + \hat{\sigma}_z)\right] + \omega_2\left[\hat{n}_2 + \frac{k_2}{2}(I - \hat{\sigma}_z)\right] - \frac{1}{2}(k_1\omega_1 + k_2\omega_2)I \\ &\quad + \frac{\Delta}{2}\hat{\sigma}_z + \beta_1\hat{n}_1|g\rangle\langle g| + \beta_2\hat{n}_2|e\rangle\langle e| + \lambda\left(\hat{a}_1^{\dagger k_1}\hat{a}_2^{k_2}\hat{\sigma}_-\right. \\ &\quad \left. + \hat{a}_1^{k_1}\hat{a}_2^{\dagger k_2}\hat{\sigma}_+\right), (\hbar = 1)\end{aligned} \tag{3}$$

where the detuning parameter $\Delta = \omega_0 - k_1\omega_1 + k_2\omega_2$, $\hat{a}_j(\hat{a}_j^\dagger)$ and $\hat{n}_j = \hat{a}_j^\dagger\hat{a}_j$ are the annihilation (creation) and number operators for the *jth* mode, β_1 and β_2 are parameters describing the intensity-dependent Stark shifts of the two levels that are due to the virtual transition to the intermediate relay level, λ is the particle-field coupling constant, ω_1 and ω_2 are the field frequencies for the two modes, ω_0 is the transition frequency of the particle (atom or trapped ion), $\hat{\sigma}_z$ is the population inversion operator, and $\hat{\sigma}_\pm$ are the 'spin flip' operators that satisfy the relation $[\hat{\sigma}_+, \hat{\sigma}_-] = \hat{\sigma}_z$ and $[\hat{\sigma}_z, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$.

Now, we look for the exact solution for the density operator $\hat{\rho}(t)$ of the Milburn Eq. (2) taking into account the Hamiltonian (3).

For convenience, we introduce three auxiliary superoperators (Chen and Kuang, 1994; Hessian, 2002; Kuang *et al.*, 1995; Kuang and Chen, 1994; Moya–Cessa *et al.*, 1993; Obada *et al.*, 1998, 1999a) $\hat{J}$, $\hat{S}$, and $\hat{L}$ defined by

$$\exp(\hat{J}\tau)\hat{\rho}(t) = \sum_{k=1}^{\infty} \frac{1}{k!} \left(\frac{\tau}{\gamma}\right)^k \hat{H}^k \hat{\rho}(t) \hat{H}^k \tag{4}$$

$$\exp(\hat{S}\tau)\hat{\rho}(t) = \exp(-i\hat{H}\tau)\hat{\rho}(t) \exp(i\hat{H}\tau) \tag{5}$$

$$\exp(\hat{L}\tau)\hat{\rho}(t) = \exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right]\hat{\rho}(t) \exp\left[-\frac{\tau}{2\gamma}\hat{H}^2\right] \tag{6}$$

where the Hamiltonian $\hat{H}$ is given by Eq. (3).

It is straightforward to obtain the formal solution of the Milburn Eq. (2) as follows:

$$\hat{\rho}(t) = \exp(\hat{J}t) \exp(\hat{S}t) \exp(\hat{L}t)\hat{\rho}(0) \tag{7}$$

where $\hat{\rho}(0)$ is the density operator of the initial particle-field system.

We assume that the initial two modes of the field inside the cavity are in superposition states and the particle in its excited state $|e\rangle$, so that:

$$\begin{aligned}\hat{\rho}(0) = \frac{1}{A}[&|\alpha_1, \alpha_2\rangle\langle\alpha_1, \alpha_2| + r^2| - \alpha_1, -\alpha_2\rangle\langle-\alpha_1, -\alpha_2| \\ &+r(|\alpha_1, \alpha_2\rangle\langle-\alpha_1, -\alpha_2| + | - \alpha_1, -\alpha_2\rangle\langle-\alpha_1, -\alpha_2|)] \\ &\times \otimes |e\rangle\langle e|\end{aligned} \tag{8}$$

where $A = [1 + r^2 + 2r \exp(-2(\alpha_1^2 + \alpha_2^2))]$, and $\alpha_j (j = 1, 2)$ are real. The parameter r can assume the values $-1, 0,$ and 1, which corresponds to an odd coherent state, a coherent state, and an even coherent state, respectively. As we know, because of the interference term in Eq. (8) it has a rapid decay to a SM when we include dissipation, so we want to see how different would be the behavior of the system if the input states are statistical mixture of the states $|\alpha_1, \alpha_2 >$ and $| - \alpha_1, -\alpha_2 >$, i.e.,

$$\hat{\rho}(0) = \frac{1}{2}[|\alpha_1, \alpha_2\rangle\langle\alpha_1, \alpha_2| + | - \alpha_1, -\alpha_2\rangle\langle-\alpha_1, -\alpha_2|] \otimes |e\rangle\langle e| \tag{9}$$

with $|\alpha_1, \alpha_2\rangle = |\alpha_1 \otimes |\alpha_2\rangle$ defined by

$$|\alpha_1, \alpha_2\rangle = \sum_{n_1,n_2=0}^{\infty} Q_{n_1} Q_{n_2} |n_1, n_2\rangle = \sum_{n_1,n_2=0}^{\infty} Q_{n_1} Q_{n_2} |n_1\rangle \otimes |n_2\rangle \tag{10}$$

where $Q_{n_j} = e^{-\alpha_j^2/2} \frac{\alpha_j^{n_j}}{\sqrt{n_j!}}$, $(j = 1, 2)$.

In a two-dimensional basis for the particle the Hamiltonian (3) can be expressed as a sum of $(\hat{H}_\circ)$, which is diagonal in this basis and $(\hat{H}_I)$, which is not. It is easy to prove that $(\hat{H}_\circ)$ and $(\hat{H}_I)$ commute, i.e.

$$[\hat{H}_\circ, \hat{H}_I] = 0. \tag{11}$$

Thus, the representation now takes the form

$$\hat{H}_\circ = \begin{bmatrix} \hat{W}(\hat{n}_1 + k_1, \hat{n}_2) + \hat{\delta}_+(\hat{n}_1 + k_1, \hat{n}_2) & 0 \\ 0 & \hat{W}(\hat{n}_1, \hat{n}_2 + k_2) + \hat{\delta}_+(\hat{n}_1, \hat{n}_2 + k_2) \end{bmatrix} \tag{12}$$

$$\hat{H}_I = \lambda \begin{bmatrix} \left[\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n_1 + k_1, n_2)\right] & \hat{a}_1^{k_1} \hat{a}_2^{\dagger k_2} \\ \hat{a}_1^{\dagger k_1} \hat{a}_2^{k_2} & -\left[\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n_1, n_2 + k_2)\right] \end{bmatrix} \tag{13}$$

with

$$\hat{W}(n_1, n_2 + k_2) = \omega_1 \hat{n}_1 + \omega_2(\hat{n}_2 + k_2),\ \hat{\delta}_\pm(n_1, n_2 + k_2)$$
$$= \frac{1}{2}[\beta_2(\hat{n}_2 + k_2) \pm \beta_1 \hat{n}_1]. \tag{14}$$

Similarly, the square of the Hamiltonian (3) can also be expressed as a sum of two matrices in the form

$$\hat{H}^2 = \hat{A} + \hat{B} \quad [\hat{A}, \hat{B}] = 0 \tag{15}$$

where $\hat{A}$ is diagonal in the form

$$\hat{A} = \begin{bmatrix} \hat{\Theta}^2(n_1 + k_1, n_2) & 0 \\ 0 & \hat{\Theta}^2(n_1, n_2 + k_2) \end{bmatrix} \tag{16}$$

and $\hat{B}$ is given by

$$\hat{B} = 2\lambda \begin{bmatrix} \hat{\eta}(n_1 + k_1, n_2)\hat{\zeta}(n_1 + k_1, n_2) & \hat{a}_1^{k_1} \hat{a}_2^{\dagger k_2} \hat{\zeta}(n_1, n_2 + k_2) \\ \hat{\zeta}(n_1, n_2 + k_2)\hat{a}_1^{k_1} \hat{a}_2^{\dagger k_2} & -\hat{\eta}(n_1, n_2 + k_2)\hat{\zeta}(n_1, n_2 + k_2) \end{bmatrix} \tag{17}$$

with

$$\hat{\eta}(n_1, n_2 + k_2) = \left[\frac{\Delta}{2\lambda} + \frac{1}{\lambda}\hat{\delta}_-(n_1, n_2 + k_2)\right],\ \hat{\zeta}(n_1, n_2 + k_2)$$

$$= [\hat{W}(n_1, n_2 + k_2) + \hat{\delta}_+(n_1, n_2 + k_2)] \tag{18}$$

$$\hat{\mu}^2(n_1, n_2 + k_2) = \hat{\eta}^2(n_1, n_2 + k_2) + \hat{\nu}^2(n_1, n_2 + k_2) \tag{19}$$

$$\hat{\nu}^2(n_1, n_2 + k_2) = \hat{a}_1^{\dagger k_1} \hat{a}_1^{k_1} \hat{a}_2^{k_2} \hat{a}_2^{\dagger k_2} = \frac{\hat{n}_1!}{(\hat{n}_1 - k_1)!} \frac{(\hat{n}_2 + k_2)!}{\hat{n}_2!} \tag{20}$$

and

$$\hat{\Theta}^2(n_1, n_2 + k_2) = \hat{\zeta}^2(n_1, n_2 + k_2) + \lambda^2 \hat{\mu}^2(n_1, n_2 + k_2). \tag{21}$$

For convenience, we introduce the auxiliary operator $\hat{\rho}_2(t)$ defined by

$$\begin{aligned} \hat{\rho}_2(t) &= \exp(\hat{S}t) \exp(\hat{L}t)\hat{\rho}(0) \\ &= \exp(-i\hat{H}_I t) \exp\left(-\frac{t}{2\gamma}\hat{B}\right) \hat{\rho}_1(t) \exp\left(-\frac{t}{2\gamma}\hat{B}\right) \exp(i\hat{H}_I t). \end{aligned} \tag{22}$$

The auxiliary operator $\hat{\rho}_1(t)$ for the initial condition (Eq. 9) defined by:

$$\hat{\rho}_1(t) = \begin{bmatrix} |\hat{\Psi}^+(t)\rangle\langle\hat{\Psi}^+(t)| + |\hat{\Psi}^-(t)\rangle\langle\hat{\Psi}^-(t)| & 0 \\ 0 & 0 \end{bmatrix} \tag{23}$$

where

$$\begin{aligned} |\hat{\Psi}^\pm(t)\rangle &= \frac{1}{\sqrt{2}} \exp\left[-\frac{t}{2\gamma}\hat{\Theta}^2(n_1 + k_1, n_2)\right] \\ &\quad \times \exp[-i\zeta(n_1 + k_1, n_2)t] \mid \pm\alpha_1, \pm\alpha_2\rangle. \end{aligned} \tag{24}$$

While for the initial (Eq. 8) the operator $\hat{\rho}_1(t)$ defined by:

$$\hat{\rho}_1(t) = \begin{bmatrix} [\mid \hat{\Psi}^+(t)\rangle + r \mid \hat{\Psi}^-(t)\rangle][\langle\hat{\Psi}^+(t) \mid +r\langle\hat{\Psi}^-(t)|] & 0 \\ 0 & 0 \end{bmatrix} \tag{25}$$

with

$$\begin{aligned} |\hat{\Psi}^\pm(t)\rangle &= \frac{1}{\sqrt{A}} \exp\left[-\frac{t}{2\gamma}\hat{\Theta}^2(n_1 + k_1, n_2)\right] \\ &\quad \times \exp[-i\hat{\zeta}(n_1 + k_1, n_2)t] \mid \pm\alpha_1, \pm\alpha_2\rangle. \end{aligned} \tag{26}$$

The powers of the operator $\hat{B}$ can be written as

$$\hat{B}^{2k} = \begin{bmatrix} [2\lambda\hat{\zeta}(n_1 + k_1, n_2)\hat{\mu}(n_1 + k_1, n_2)]^{2k} & 0 \\ 0 & [2\lambda\hat{\zeta}(n_1, n_2 + k_2)\hat{\mu}(n_1, n_2 + k_2)]^{2k} \end{bmatrix} \tag{27}$$

$$\hat{B}^{2k+1} = \begin{bmatrix} \hat{\eta}_1 \frac{[2\lambda\hat{\zeta}_1\hat{\mu}_1]^{2k+1}}{\hat{\mu}_1} & \hat{a}_1^{k_1}\hat{a}_2^{k_2} \frac{[2\lambda\hat{\zeta}_2\hat{\mu}_2]^{2k+1}}{\hat{\mu}_2} \\ \frac{[2\lambda\hat{\zeta}_2\hat{\mu}_2]^{2k+1}}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & -\hat{\eta}_2 \frac{[2\lambda\hat{\zeta}_2\hat{\mu}_2]^{2k+1}}{\hat{\mu}_2} \end{bmatrix} \tag{28}$$

then we can write the operator $\exp[-\frac{t}{2\gamma}\hat{B}]$ in the form

$$\exp\left[-\frac{t}{2\gamma}\hat{B}\right] = \begin{bmatrix} \hat{X}_1(t) - \hat{\eta}_1\frac{\hat{Y}_1(t)}{\hat{\mu}_1} & -\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{Y}_2(t)}{\hat{\mu}_2} \\ -\frac{\hat{Y}_2(t)}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{X}_2(t) + \hat{\eta}_2\frac{\hat{Y}_2(t)}{\hat{\mu}_2} \end{bmatrix} \tag{29}$$

where

$$\hat{X}_2(t) = \cosh\left[\frac{\lambda t}{\gamma}\hat{\zeta}_2\hat{\mu}_2\right], \qquad \hat{Y}_2(t) = \sinh\left[\frac{\lambda t}{\gamma}\hat{\zeta}_2\hat{\mu}_2\right]. \tag{30}$$

Similarly, we can write the operator $\exp[-i\hat{H}_I t]$ in the two-dimensional basis for the particle as

$$\exp[-i\hat{H}_I t] = \begin{bmatrix} \hat{C}_1(t) - i\hat{\eta}_1\frac{\hat{S}_1(t)}{\hat{\mu}_1} & -i\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{S}_2(t)}{\hat{\mu}_2} \\ i\frac{\hat{S}_2(t)}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{C}_2(t) + i\hat{\eta}_2\frac{\hat{S}_2(t)}{\hat{\mu}_2} \end{bmatrix} \tag{31}$$

with

$$\hat{C}_2(t) = \cos[\lambda t\hat{\mu}(n_1, n_2 + k_2)] \quad \text{and} \quad \hat{S}_2(t) = \sin[\lambda t\hat{\mu}(n_1, n_2 + k_2)]. \tag{32}$$

Then,

$$\exp[-i\hat{H}_I t]\,\exp\left(-\frac{t}{2\gamma}\hat{B}\right) = \begin{bmatrix} \hat{R}_1(t) - \eta_1\frac{\hat{V}_1(t)}{\hat{\mu}_1} & -\hat{a}_1^{k_1}\hat{a}_2^{k_2}\frac{\hat{V}_2(t)}{\hat{\mu}_2} \\ -\frac{\hat{V}_2(t)}{\hat{\mu}_2}\hat{a}_1^{\dagger k_1}\hat{a}_2^{\dagger k_2} & \hat{R}_2(t) + \eta_2\frac{V_2(t)}{\hat{\mu}_2} \end{bmatrix} \tag{33}$$

where

$$\hat{R}_2(t) = \hat{C}_2(t)\hat{X}_2(t) + i\hat{S}_2(t)\hat{Y}_2(t) \quad \hat{V}_2(t) = \hat{C}_2(t)\hat{Y}_2(t) + i\hat{S}_2(t)\hat{X}_2(t). \tag{34}$$

Note that in the above Eqs. (28–34), we have used the subscript 1 instead of $(n_1 + k_1, n_2)$ and 2 instead of $(n_1, n_2 + k_2)$.

Now, we can obtain an explicit expression for the operator $\hat{\rho}_2(t)$ for the two initial (9) and (8) as follows:

Substituting Eqs. (23) and (33) into Eq. (22), we obtain an explicit expression of the operator $\hat{\rho}_2(t)$ for the initial (9) as follows:

$$\hat{\rho}_2(t) = \begin{bmatrix} \hat{\Psi}_{11}^{+}(t) + \hat{\Psi}_{11}^{-}(t) & \hat{\Psi}_{12}^{+}(t) + \hat{\Psi}_{12}^{-}(t) \\ \hat{\Psi}_{21}^{+}(t) + \hat{\Psi}_{21}^{-}(t) & \hat{\Psi}_{22}^{+}(t) + \hat{\Psi}_{22}^{-}(t) \end{bmatrix} \tag{35}$$

where we have used the following symbol

$$\hat{\Psi}_{ij}^{\pm}(t) = |\hat{\Psi}_i^{\pm}(t)\rangle\langle\hat{\Psi}_{ij}^{\pm}(t)|(i, j = 1, 2) \tag{36}$$

with

$$|\hat{\Psi}_1^{\pm}(t)\rangle = \left[\hat{R}(n_1 + k_1, n_2, t) - \frac{\Delta}{2\lambda}\frac{\hat{V}(n_1 + k_1, n_2, t)}{\hat{\mu}(n_1 + k_1, n_2)}\right]|\hat{\Psi}^{\pm}(t)\rangle \tag{37}$$

$$|\hat{\Psi}_2^{\pm}(t)\rangle = \left[-\hat{a}_1^{\dagger k_1} \hat{a}_2^{\dagger k_2} \frac{\hat{V}(n_1 + k_1, n_2, t)}{\hat{\mu}(n_1 + k_1, n_2)} \right] |\hat{\Psi}^{\pm}(t)\rangle \tag{38}$$

where $|\hat{\psi}^{\pm}(t) >$ is given by Eq. (24). Also by substituting Eqs. (25) and (33) into Eq. (22), we obtain an explicit expression of the operator $\hat{\rho}_2(t)$ for the initial (8) as follows:

$$\hat{\rho}_2(t) = \begin{bmatrix} \hat{\Psi}_{11}(t) & \hat{\Psi}_{12}(t) \\ \hat{\Psi}_{21}(t) & \hat{\Psi}_{22}(t) \end{bmatrix} \tag{39}$$

where we have used the following symbol

$$\hat{\Psi}_{ij}(t) = |\hat{\Psi}_i(t)\rangle\langle\hat{\Psi}_j(t)|(i, j = 1, 2) \tag{40}$$

with

$$|\hat{\Psi}_1(t)\rangle = \left[\hat{R}(n_1 + k_1, n_2, t) - \hat{\eta}_1 \frac{\hat{V}(n_1 + k_1, n_2, t)}{\hat{\mu}(n_1 + k_1, n_2)} \right] \times [|\hat{\Psi}^{+}(t)\rangle + r|\hat{\Psi}^{-}(t)\rangle] \tag{41}$$

$$|\hat{\Psi}_2(t)\rangle = \left[-a_1^{\dagger k_1} a_2^{k_2} \frac{\hat{V}(n_1 + k_1, n_2, t)}{\hat{\mu}(n_1 + k_1, n_2)} \right] \times [|\hat{\Psi}^{+}(t)\rangle + r|\hat{\Psi}^{+}(t)\rangle + r|\hat{\Psi}^{-}(t)\rangle] \tag{42}$$

where $|\hat{\Psi}^{\pm}(t)\rangle$ is given by Eq. (26). By using the definition of the superoperator $\hat{J}$, it is straightforward to obtain the action of the operator $\exp(\hat{J}t)$ on the density operator $\hat{\rho}_2(t)$ as follows (Chen and Kuang, 1994; Hessian, 2002; Kuang *et al.*, 1995; Kuang and Chen, 1994; Moya–Cessa *et al.*, 1993; Obada *et al.*, 1998, 1999a):

$$\hat{\rho}(t) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{t}{\gamma} \right)^k \hat{H}^k \hat{\rho}_2(t) \hat{H}^k = \begin{bmatrix} \hat{\rho}_{11}(t) & \hat{\rho}_{12}(t) \\ \hat{\rho}_{21}(t) & \hat{\rho}_{22}(t) \end{bmatrix} \tag{43}$$

with

$$\hat{\rho}_{ij}(t) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{t}{\gamma} \right)^k \hat{M}_{ij}^{(k)}(t) \tag{44}$$

where the Hamiltonian $\hat{H}$ is given by Eq. (3) and the operator $\hat{\rho}_2$ is given by Eqs. (35) and (39) for SM and SS, respectively. Making use of this solution, we can evaluate mean values of operators of interest. In what follows, we will use it to study the influence of the intrinsic decoherence on dynamics of the particle (atom or trapped ion) and the cavity field in the JCM.

3. INFLUENCE OF THE INTRINSIC DECOHERENCE ON NONCLASSICAL PROPERTIES OF THE SYSTEM

In this section, we investigate the influence of the intrinsic decoherence on nonclassical properties of the particle and the field for the multiquanta two-mode JCM in the presence of Stark shift with a detuning parameter in either a superposition state (SS) or a mixture state (SM).

3.1. Population Inversion

It is well known that in the JCM the quantum coherences which are built up during the interaction between the field and the particle significantly affect the dynamics of the particle. The existence of the quantum coherences is the reason why one can observe collapses and revivals of the population inversion of the particle. Now we evaluate the population inversion in the multiquanta JCM. The population inversion is defined as the expectation value of the operator $\hat{\sigma}_z$, i.e.

$$\langle \hat{\sigma}_z(t) \rangle = Tr[\hat{\rho}(t)\hat{\sigma}_z]. \tag{45}$$

By using Eqn. (43), we can express Eqn. (45) in the following form:

$$\langle \hat{\sigma}_z(t) \rangle = \sum_{n_1,n_2,k=0}^{\infty} \frac{1}{k!} \left(\frac{t}{\gamma}\right)^k \Big\{ \langle n_1, n_2 \mid \hat{M}_{11}^{(k)}(t) \mid n_1, n_2 \rangle - \langle n_1, n_2 \mid \hat{M}_{22}^{(k)}(t) \mid n_1, n_2 \rangle \Big\}. \tag{46}$$

If the field is initially prepared in a SM of states $\mid \alpha_1, \alpha_2 \rangle$ and $\mid -\alpha_1, -\alpha_2 \rangle$ (Eq. 9), the population inversion will be:

$$W_M(t) = \sum_{n_1,n_2=0}^{\infty} \frac{|Q_{n1}|^2 |Q_{n2}|^2}{\mu^2(n_1+k_1,n_2)} \Big\{ \eta^2(n_1+k_1,n_2) + \nu^2(n_1+k_1,n_2) \times \exp\left[-\frac{2\lambda^2 t}{\gamma}\mu^2(n_1+k_1,n_2)\right] \cos 2\lambda t \mu(n_1+k_1,n_2) \Big\}. \tag{47}$$

As we expected, the population inversion is the same as if the input field was a coherent state. However, if the field is initially prepared in a superposition of coherent states SS (Eq. 8), the population inversion will be:

$$W_S(t) = \sum_{n_1,n_2=0}^{\infty} \frac{|Q_{n1}|^2 |Q_{n2}|^2}{\mu^2(n_1+k_1,n_2)} \frac{[1+r(-1)^{n1+n2}]^2}{\left[1+r^2+2r\ \exp\left(-2\left(\alpha_1^2+\alpha_2^2\right)\right)\right]} \Big\{ \eta^2(n_1+k_1,n_2) + \nu^2(n_1+k_1,n_2) \exp\left[-\frac{2\lambda^2 t}{\gamma}\mu^2(n_1+k_1,n_2)\right] \cos 2\lambda t \mu(n_1+k_1,n_2) \Big\}. \tag{48}$$

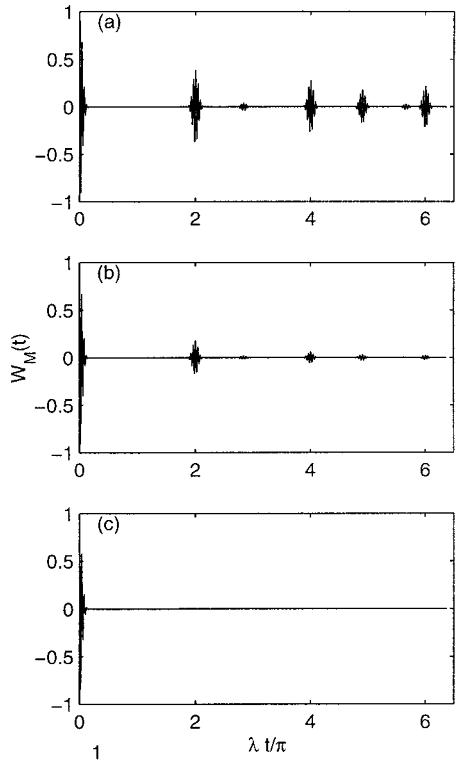

Fig. 1. Population inversion $\langle\sigma_z(t)\rangle$ as a function of scaled time $\lambda t/\pi$ of the particle initially prepared in the excited state and the field initially prepared in a statistical mixture of coherent states $|\alpha_1, \alpha_2\rangle$ and $|-\alpha_1, -\alpha_2\rangle(\overline{n}_1 = \overline{n}_2 = 25)$ for various values of the parameter $\frac{\lambda}{\gamma}$: (a) $\frac{\lambda}{\gamma} = 10^{-6}$, (b) $\frac{\lambda}{\gamma} = 5 \times 10^{-5}$ and (c) $\frac{\lambda}{\gamma} = 10^{-4}$.

Now, we discuss the general behavior of the population inversion for the multi-quanta JCM, when the particle (atom or trapped ion) initially starts in the excited state and the field initially in a mixture state SM or a superposition of coherent states SS (even coherent state (r = 1)).

The numerical results are shown in Figs. 1–6, for various values of the decoherence parameter $\frac{\lambda}{\gamma}$, and different values of the Stark shift parameter and fixed initial mean numbers of quanta $\overline{n}_1$ and $\overline{n}_2$ for two quanta ($k_1 = k_2 = 1$).

In Figs. 1 and 2, we plot the population inversion (in the absence of Stark shift ($\beta_1/\lambda = \beta_2/\lambda = 0$)) for three values of the parameter $\frac{\lambda}{\gamma}$ with the fixed initial mean numbers of quanta $\overline{n}_1$ and $\overline{n}_2$ in the case of the exact resonance, i.e., the

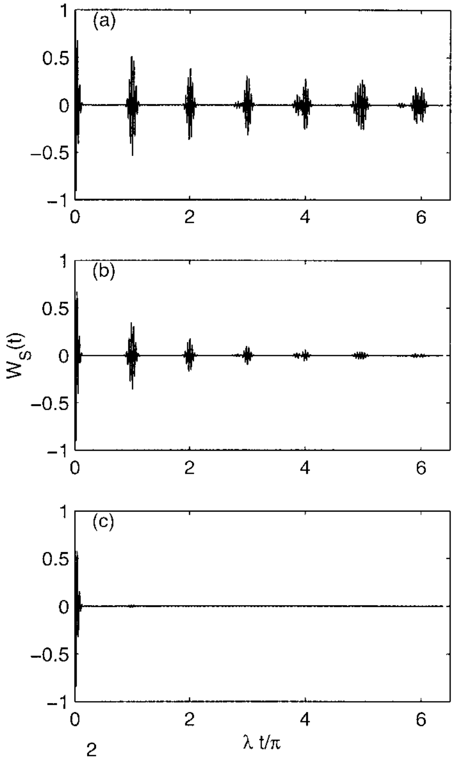

Fig. 2. The same as in Fig. 1 but with the field initially prepared in a superposition state (even coherent state).

detuning parameter $\frac{\Delta}{2\lambda} = 0$, when the field initially in a mixture state SM and in a superposition of coherent states SS (even coherent state ($r = 1$)), respectively. We see that for a superposition of coherent states SS (even coherent state ($r = 1$) (see Fig. 2), the revival time will be approximately half of the revival time for the SM. This, in effect, due to the interference between the two coherent states in the superposition, and can be understood looking at the photon number distribution of the initial fields.

In Figs. 3 and 4 and Figs. 5 and 6, we plotted the population inversion in the presence of Stark shift ($\beta_1/\lambda = \beta_2/\lambda = 1.0$) and ($\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$) respectively.

In the case described in Figs. 3 and 4, the Stark shift parameter is ($\beta_1/\lambda = \beta_2/\lambda = 1.0$), which corresponds to the case in which the two levels of the particle are equally strongly coupled with the intermediate relay level. From these figures,

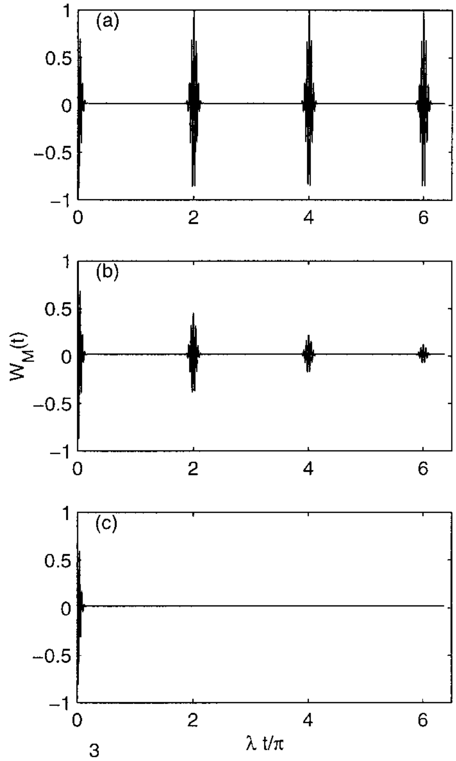

Fig. 3. The same as in Fig. 1 but in the presence of Stark shift $\beta_1/\lambda = \beta_2/\lambda = 1.0$.

we see the Stark shift leads to increase of the values of the atomic revivals of the population inversion (see Figs. 3 and 4).

In Figs. 5 and 6, we show the cases in which the two levels have unequal Stark shift ($\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$). We see the Stark shift leads to decrease of the values of the atomic revivals of the population inversion (see Figs. 5 and 6).

Also, these figures show that with the decrease of the decoherence parameter γ, i.e., with a more rapid suppression of quantum coherence we can observe rapid deterioration of revivals of the population inversion. This means that the decay of quantum coherence is due to the very specific time evolution described by the Milburn Eq. (2), i.e., due to the intrinsic decoherence

Obviously, when $\gamma \to \infty$, the population inversion reduces to the well-known expression for the population inversion in the off-resonant nondegenerate multi-photon JCM governed by the von Neumann equation.

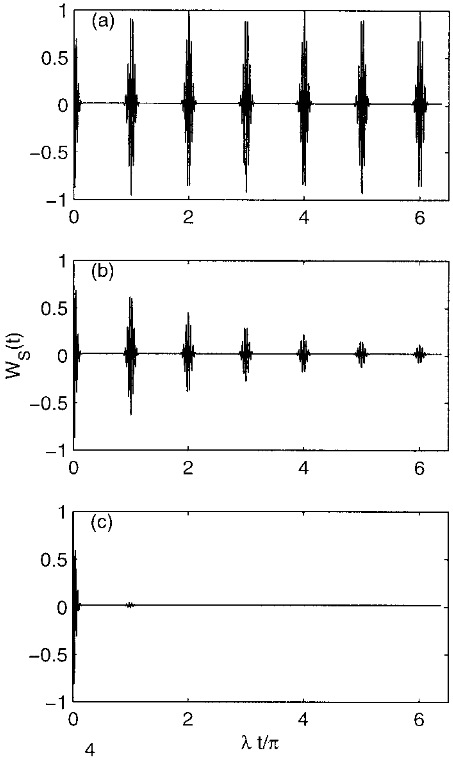

Fig. 4. The same as in Fig. 2 but in the presence of Stark shift $\beta_1/\lambda = \beta_2/\lambda = 1.0$.

3.2. Oscillations of the Number Distribution

It is known that oscillations of the number distribution of the quanta in the JCM are a kind of the nonclassical effects of the cavity field. To see the influence of intrinsic decoherence on this kind of nonclassical properties, we discuss statistics in the field modes. The reduced density operator of the cavity field can be obtained by taking the trace of the total density operator $\hat{\rho}(t)$ over the atomic states, that is, $\hat{\rho}_F = Tr_A\hat{\rho}(t)$. Then the probability distribution function for finding n_j quanta in the *jth* mode is calculated from the formula

$$P(n_1, n_2 + k_2; t) = \sum_{n_1, n_2, k=0}^{\infty} \frac{1}{k!}\left(\frac{t}{\gamma}\right)\left\{\langle n_1, n_2 \mid \hat{M}_{11}^{(k)}(t) \mid n_1, n_2\rangle + \langle n_1, n_2 \mid \hat{M}_{22}^{(k)}(t) \mid n_1, n_2\rangle\right\}. \quad (49)$$

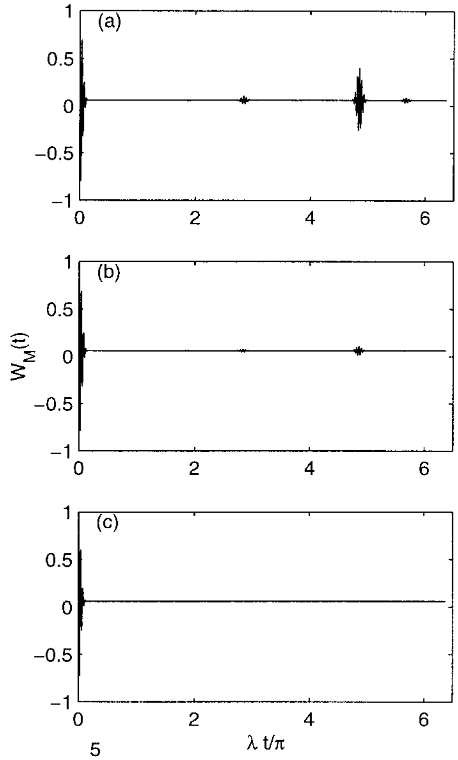

Fig. 5. The same as in Fig. 1 but with the presence of Stark shift $\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$.

If the field is initially prepared in a SM of states (Eq. 9), we find that

$$
\begin{aligned}
P_M(n_1, n_2 + k_2, t) = \langle n_1, n_2 \mid \hat{\rho}_F(t) \mid n_1, n_2 \rangle = \frac{1}{2} \mid Q_{n_1} \mid^2 \mid Q_{n_2} \mid^2 \\
\times \left\{ 1 + \frac{\eta^2(n_1 + k_1, n_2)}{\mu^2(n_1 + k_1, n_2)} + \frac{\nu^2(n_1 + k_1, n_2)}{\mu^2(n_1 + k_1, n_2)} \right. \\
\left. \times \exp\left[-\frac{2\lambda^2 t}{\gamma} \mu^2(n_1 + k_1, n_2) \right] \cos 2\lambda t \mu(n_1 + k_1, n_2) \right\} \\
+ \frac{1}{2} |Q_{n_1 - k_1}|^2 |Q_{n_2 - k_2}|^2 \left\{ 1 - \frac{\eta^2(n_1, n_2 + k_2)}{\mu^2(n_1, n_2 + k_2)} \right. \\
- \frac{\nu^2(n_1, n_2 + k_2)}{\mu^2(n_1, n_2 + k_2)} \exp\left[-\frac{2\lambda^2 t}{\gamma} \mu^2(n_1, n_2 + k_2) \right] \\
\left. \times \cos 2\lambda t \mu(n_1, n_2 + k_2) \right\}
\end{aligned}
\tag{50}
$$

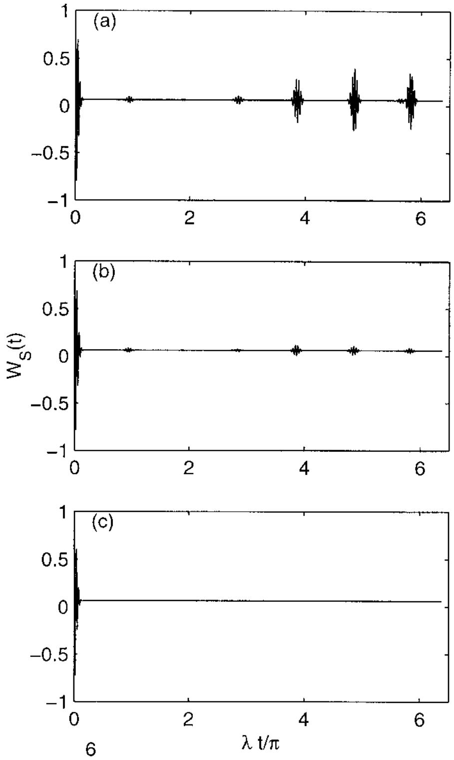

Fig. 6. The same as in Fig. 2 but with the presence of Stark shift $\beta_1/\lambda = 0.5$, $\beta_2/\lambda = 1.0$.

If the field is initially prepared in a superposition of coherent states (Eq. 8), we find:

$$
\begin{aligned}
P_S(n_1, n_2 + k_2; t) = \frac{1}{2} \mid Q_{n_1} \mid^2 \mid Q_{n_2} \mid^2 & \frac{[1 + r(-1)^{n_1+n_2}]^2}{\left[1 + r^2 + 2r \, \exp\left(-2\left(\alpha_1^2 + \alpha_2^2\right)\right)\right]} \\
& \times \left\{ 1 + \frac{\eta^2(n_1 + k_1, n_2)}{\mu^2(n_1 + k_1, n_2)} + \frac{\nu^2(n_1 + k_1, n_2)}{\mu^2(n_1 + k_1, n_2)} \right. \\
& \times \left. \exp\left[-\frac{2\lambda^2 t}{\gamma} \mu^2(n_1 + k_1, n_2) \right] \cos 2\lambda t \mu(n_1 + k_1, n_2) \right\} \\
& + \frac{1}{2} \mid Q_{n_1 - k_1} \mid^2 \mid Q_{n_2 + k_2} \mid^2 \frac{[1 + r(-1)^{n_1 - k_1 + n_2 - k_2}]^2}{\left[1 + r^2 + 2r \exp\left(-2\left(\alpha_1^2 + \alpha_2^2\right)\right)\right]}
\end{aligned}
$$

$$\times \left\{ 1 - \frac{(\eta_2)^2}{\mu^2(n_1, n_2 + k_2)} - \frac{\nu^2(n_1, n_2 + k_2)}{\mu^2(n_1, n_2 + k_2)} \right.$$
$$\left. \times \exp\left[-\frac{2\lambda^2 t}{\gamma} \mu^2(n_1, n_2 + k_2) \right] \cos 2\lambda t \mu(n_1, n_2 + k_2) \right\} \quad (51)$$

where $|Q_{n_1}|^2$ and $|Q_{n_2}|^2$ are the initial values for the distribution function given by Eq. (12). We can use the time-dependent number distribution that are obtained in Eqs. (50 and 51) to evaluate some quantities relevant to the field statistics. For example, the mean number of quanta in the *ith* mode are found to be for the two cases:

$$\langle n_i(t) \rangle_M = \bar{n}_i + \frac{k_i}{2} \sum_{n_1, n_2 = 0}^{\infty} |\, Q_{n_1} \,|^2 |\, Q_{n_2} \,|^2 \, \frac{\nu^2(n_1 + k_1, n_2)}{\mu^2(n_1 + k_1, n_2)}$$
$$\times \left(1 - \exp\left[-\frac{2\lambda^2 t}{\gamma} \mu^2(n_1 + k_1, n_2) \right] \right.$$
$$\left. \times \cos 2\lambda t \mu(n_1 + k_1, n_2) \right) \quad (52)$$

and

$$\langle n_i(t) \rangle s = \bar{n}_i + \frac{k_i}{2} \sum_{n_1, n_2 = 0}^{\infty} |\, Q_{n_1} \,|^2 |\, Q_{n_2} \,|^2 \, \frac{[1 + r(-1)^{n_1 + n_2}]^2}{\left[1 + r^2 + 2r \exp\left(-2\left(\alpha_1^2 + \alpha_2^2\right)\right)\right]}$$
$$\times \left\{ \frac{\nu^2(n_1 + k_1, n_2)}{\mu^2(n_1 + k_1, n_2)} \left(1 - \exp\left[-\frac{2\lambda^2 t}{\gamma} \mu^2(n_1 + k_1, n_2) \right] \right. \right.$$
$$\left. \left. \times \cos 2\lambda t \mu(n_1 + k_1, n_2) \right) \right\} \quad (53)$$

From the above expressions, we see that the intrinsic decoherence term in the Milburn Eq. (2) leads to the appearance of the decay factors $\exp[-\frac{2\lambda^2 t}{\gamma}\mu^2(n_1 + k_1, n_2)]$ in Eqs. (50–53), which are responsible for the weakening of the oscillatory behavior of the mean number of quanta in the field, where it weakened with the decrease of the intrinsic decoherence parameter γ.

Obviously, when $\gamma \to \infty$, Eqs. (50–53) reduce to the usual expressions of the oscillations of the number distribution and the intensity of the cavity field governed by the Schrödinger dynamics.

4. CONCLUSIONS

In this article, we have studied the two-mode JCM governed by the Milburn equation in the presence of Stark shift with a detuning parameter in either a superposition state (SS) or a mixture state (SM). An analytic solution for the

Milburn equation for the multiquanta model has been obtained. The density operator is then used to study the influence of the intrinsic decoherence and Stark shift on nonclassical properties in the JCM, such as population inversion and photon number statistics. It is shown that intrinsic decoherence suppresses nonclassical effects of the cavity field in the JCM. Also, we see the Stark shift leads to increase of the values of the atomic revivals of the population inversion in SM and SS when the two levels of the particle are equally strongly coupled with the intermediate relay level, while it leads to decrease of the values of the atomic revivals of the population inversion when the two levels have unequal Stark shift.

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